# GROWTH RATE OF LIQUID DROPS ON A FLAT SURFACE DURING DROPWISE CONDENSATION OF VAPOR FROM A VAPOR – GAS MIXTURE

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The growth rate of drops on a flat surface during condensation of a vapor from a vapor–gas mixture is determined theoretically and experimentally.

Considerable interest is presently being displayed in studies of the process of heat and mass exchange during the dropwise condensation of vapor from vapor–gas mixtures onto solid surfaces. However, the principles of the formation and growth of drops of condensate on cooled surfaces have been insufficiently studied. The study of the growth rate of liquid drops on a surface has been conducted only with condensation in a medium of saturated vapor [1], i.e., under conditions when the growth rate of the drops is limited only by the thermal resistance of the drops of condensate. During vapor condensation from a vapor–gas mixture with a high concentration of the noncondensing component, the growth rate of the drops is determined by the diffusion resistance to the transport of vapor to the condensation surface, while the role of thermal resistance is no longer significant in a majority of cases.

Let us consider the dropwise condensation of vapor onto the solid surface of a disk of radius $R_0$ whose temperature is kept constant ($T = T_W$) from a half-space ($z > 0$) filled with a vapor–gas medium having thermodynamic parameters which are constant in time (Fig. 1). At an arbitrary time t in a region of the condensation surface whose transverse size is small compared with the characteristic linear scale of the surface itself there will exist $N\delta s$ drops of condensate generally of different sizes (N is the number of drops per unit area of the surface; $\delta s$ is the area of the indicated surface element). It is assumed that there is a relatively large number of drops in the small element, and therefore a statistical approach is fully acceptable for the study of the kinetics of their growth.

The drops have the shape of spherical segments corresponding to a contact angle $\beta_0$ (Fig. 1b). The volume of the k-th drop is

$$\Omega_k = \frac{\pi a_k^3 (1-\cos\beta_0)^2 (2+\cos\beta_0)}{3\sin^3\beta_0}. \tag{1}$$

The specific flux of vapor to the surface element is

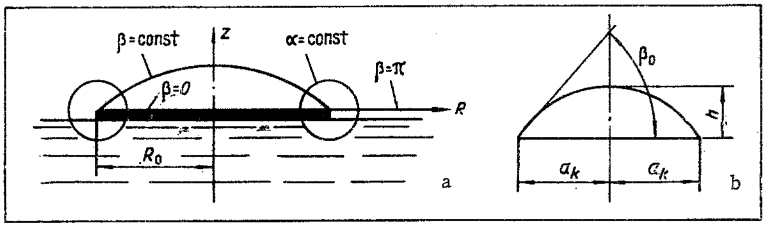


Fig. 1. Calculating diagram.

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$$j = \rho' \sum_{k=1}^{N(P,t)} \frac{d\Omega_k}{dt}. \quad (2)$$

Here P is an arbitrary point of element $\delta s$.

The distribution over the surface of the specific fluxes j of vapor is found through the solution of the corresponding diffusion problem. As yet the sizes of the drops are small compared with the characteristic linear scale L of the cooled surface, and their presence has almost no effect on the field of vapor concentrations near the surface. It is also assumed that the Grashof number corresponding to this process, constructed in accordance with the size L, is not very large and one can ignore the effect of free convection on the diffusion of vapor to the condensation surface.

Since during steady-state diffusion the partial density of the vapor must satisfy the Laplace equation ($\nabla^2\rho'' = 0$ for $z > 0$), the problem is reduced to the determination of the harmonic function $\rho''$ in the upper half-space which satisfies the following boundary conditions at the surface $z = 0$:

$$\rho'' = \rho''_w \quad \text{when} \quad R < R_0, \quad \rho'' = \rho''_\infty \quad \text{when} \quad R > R_0.$$

It is very convenient to solve such a problem in a toroidal coordinate system ($\alpha$, $\beta$, $\varphi$; see Fig. 1a) which is introduced with the help of the equations

$$R + iz = R_0 \,\text{th}\frac{\alpha + i\beta}{2}, \quad R = \frac{R_0 \,\text{sh}\,\alpha}{\text{ch}\,\alpha + \cos\beta}, \quad z = \frac{R_0 \sin\beta}{\text{ch}\,\alpha + \cos\beta}. \quad (3)$$

Since $\partial\rho''/\partial\varphi = 0$ because of symmetry, the Laplace equation in toroidal coordinates is written in the form

$$\frac{\partial}{\partial\alpha}\left(\frac{\text{sh}\,\alpha}{\text{ch}\,\alpha + \cos\beta}\,\frac{\partial\rho''}{\partial\alpha}\right) + \frac{\partial}{\partial\beta}\left(\frac{\text{sh}\,\alpha}{\text{ch}\,\alpha + \cos\beta}\,\frac{\partial\rho''}{\partial\beta}\right). \quad (4)$$

The corresponding boundary conditions are

$$\rho'' = \rho''_w \quad \text{when} \quad \beta = 0; \quad \rho'' = \rho''_\infty \quad \text{when} \quad \beta = \pi. \quad (5)$$

The solution of the problem (4)-(5) is sought in the form

$$\rho'' = \rho''_\infty + \sqrt{\text{ch}\,\alpha + \cos\beta} \int_0^\infty f(\tau)\,\text{sh}\,[\tau(\pi - \beta)]\, p_{-1/2+i\tau}(\text{ch}\,\alpha)\, d\tau, \quad (6)$$

where $p_{-1/2+i\tau}(\cosh\alpha)$ is a Legendre function with a complex subscript. The function (6) satisfies Eq. (4) and the second boundary condition of (5). Using the first boundary condition of (5) we can obtain an integral equation for the determination of the function $f(\tau)$:

$$\int_0^\infty f(\tau)\,\text{sh}(\pi\tau)\, p_{-1/2+i\tau}(\text{ch}\,\alpha)\, d\tau = -\frac{\Delta\rho''}{\sqrt{\text{ch}\,\alpha + 1}},$$

where $\Delta\rho'' = \rho''_\infty - \rho''_w$. But

$$\frac{1}{\sqrt{\text{ch}\,\alpha + 1}} = \sqrt{2}\int_0^\infty p_{-1/2+i\tau}(\text{ch}\,\alpha)\,\frac{d\tau}{\text{ch}(\pi\tau)}.$$

Thus,

$$f(\tau) = -\frac{\sqrt{2}\,\Delta\rho''}{\text{ch}(\pi\tau)\,\text{sh}(\pi\tau)}$$

and the distribution of the partial vapor density in the volume $0 < \beta < \pi$ is described by the equation

$$\rho'' = \rho''_\infty - \sqrt{2(\text{ch}\,\alpha + \cos\beta)}\,\Delta\rho'' \int_0^\infty \frac{\text{ch}\,[\tau(\pi - \beta)]}{\text{ch}(\pi\tau)\,\text{sh}(\pi\tau)}\, p_{-1/2+i\tau}(\text{ch}\,\alpha)\, d\tau. \quad (7)$$

Since the sizes of the drops of condensate and the distances between neighboring drops are small compared with the radius $R_0$ of the disk, one can determine the specific fluxes j of vapor onto the disk, averaged over a region of the surface of the disk whose transverse size is small compared with $R_0$ but in which there are a large number of drops. In the determination of these fluxes one can ignore the discrete nature of the condensation process due to the presence on the disk of individual "foci" of condensation:

$$j = \left(\frac{D}{H}\frac{\partial\rho''}{\partial\beta}\right)_{\beta=0}$$

[the Lamé constant $H = R_0/(\cosh\alpha + \cos\beta)$],

$$\left(\frac{\partial\rho''}{\partial\beta}\right)_{\beta=0} = \Delta\rho''\sqrt{2(\operatorname{ch}\alpha+1)}\int\limits_0^\infty \frac{\tau}{\operatorname{sh}(\pi\tau)}\, p_{-1/2+i\tau}(\operatorname{ch}\alpha)\,d\tau.$$

Using the integral representation [2]

$$p_{-1/2+i\tau}(\operatorname{ch}\alpha) = \frac{2}{\pi}\int\limits_0^\alpha \frac{\cos(\tau s)\,ds}{\sqrt{2(\operatorname{ch}\alpha-\operatorname{ch}s)}},$$

changing the order of integration, and considering the value of the integral [3]

$$\int\limits_0^\infty \frac{\tau\cos(\tau s)\,d\tau}{\operatorname{sh}(\pi\tau)} = \frac{1}{4\operatorname{ch}^2 s/2},$$

we obtain

$$\left(\frac{\partial\rho''}{\partial\beta}\right)_{\beta=0} = \frac{\Delta\rho''}{2\pi}\sqrt{2(\operatorname{ch}\alpha+1)}\int\limits_0^\alpha \frac{ds}{\operatorname{ch}^2(s/2)\sqrt{2(\operatorname{ch}\alpha-\operatorname{ch}s)}}.$$

But

$$\operatorname{ch}\alpha-\operatorname{ch}s = 2(\operatorname{ch}^2\alpha/2-\operatorname{ch}^2 s/2).$$

Consequently,

$$\left(\frac{\partial\rho''}{\partial\beta}\right)_{\beta=0} = \frac{\Delta\rho''}{2\pi}\sqrt{2(\operatorname{ch}\alpha+1)}\cdot\int\limits_1^{\operatorname{ch}\alpha/2} \frac{dx}{x^2\sqrt{(\operatorname{ch}^2\alpha/2-x^2)(x^2-1)}}.$$

Since [3]

$$\int\limits_1^{\operatorname{ch}\alpha/2} \frac{dx}{x^2\sqrt{(\operatorname{ch}^2\alpha/2-x^2)(x^2-1)}} = \frac{E(\operatorname{th}\alpha/2)}{\operatorname{ch}(\alpha/2)},$$

where E(x) is the complete elliptic integral

$$E(x) = \frac{\pi}{2}\left\{1-\frac{1}{2^2}x^2-\frac{1^2\cdot 3}{2^2\cdot 4^2}x^4-\ldots-\left[\frac{(2n-1)!!}{2^n n!}\right]^2\frac{x^{2n}}{2n-1}-\ldots\right\},$$

we obtain

$$j = \frac{D\Delta\rho''(\operatorname{ch}\alpha+1)E(\operatorname{th}\alpha/2)}{\pi R_0}.$$

But because of (3), when $\beta = 0$

$$\frac{2}{\operatorname{ch}\alpha+1} = 1-\frac{R^2}{R_0^2},\qquad \operatorname{th}\frac{\alpha}{2} = \frac{R}{R_0}.$$

Thus, the latter result can be represented in a form more convenient for calculations:

$$j = \frac{2D\Delta\rho'' E(R/R_0)}{\pi R_0[1-(R/R_0)^2]}. \tag{8}$$

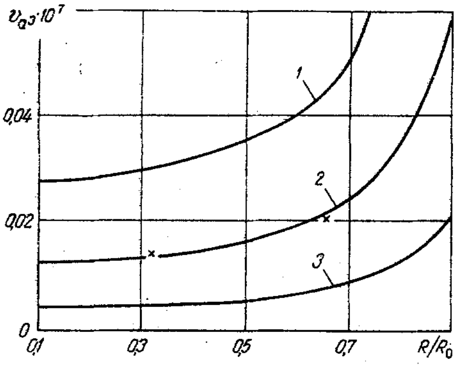


Fig. 2. Dependence of $v_{a^3} = (d/dt) \sum_{k=1}^{N(R,t)} a_k^3(R, t)$ on $R/R_0$ [according to Eq. (9)]: 1) $\Delta\rho'' = 0.048$; 2) 0.023; 3) 0.0082.

By substituting (1) and (2) into the left side of the latter equation we can find the total growth rate of drops located at a distance R from the center of the disk per unit of surface:

$$\frac{d}{dt}\sum_{k=1}^{N(R,t)} a_k^3(R, t)$$

$$= \frac{6D\Delta\rho''\sin^3\beta_0}{\pi^2\rho' R_0(1-\cos\beta_0)^2(2+\cos\beta_0)}\ \frac{E(R/R_0)}{1-R^2/R_0^2}. \tag{9}$$

It is assumed that the growth of stationary drops takes place relatively uniformly, i.e., in a narrow enough ring R, R + δR the drops have about the same radius $a = a(R, t)$ and are distributed rather uniformly within the limits of the narrow ring. Moreover, during some time interval Δt the growth of these drops scarcely leads to their merging, while their density N on the surface does not vary with time, being only a function of the distance $R[N = N(R)]$ from the center of the disk. Then in place of (9) one can write

$$\Delta a^3 = \frac{6D\Delta\rho''\sin^3\beta_0}{\pi^2\rho' R_0(1-\cos\beta_0)^2(2+\cos\beta_0)}\ \frac{E(R/R_0)}{(1-R^2/R_0^2)N(R)}\Delta t. \tag{10}$$

The dependence of the growth rate of the drops on $R/R_0$ and $\Delta\rho''$ = const according to Eq. (9) is presented in Fig. 2.

Equations (9) and (10) are valid if the conditions adopted in their derivation are satisfied. If individual drops which slightly affect one another are growing on the surface of the disk and the distances between them are comparable with the radius of the disk, then the solution of the corresponding problem, in general, becomes considerably more complicated. However, it is relatively easy to obtain an upper estimate for the growth rate of drops separated from the boundary of the disk by distances $l \gg a$. The growth rate of a single drop on a disk of infinite radius can be taken as such an estimate.

If, as in the case of "dense" covering of the surface of the disk by drops, the thermal resistance of the condensate is neglected in comparison with the diffusion resistance of the supply of vapor to it, then the determination of the field of partial densities of vapor in the vicinity of a single drop comes down to the solution of Eq. (2) with the following boundary conditions:

$$\rho'' = \rho''_w \quad \text{when}\ \beta = 0, \quad \partial\rho''/\partial\beta = 0 \ \text{when}\ \beta = \pi, \quad \rho'' = \rho''_\infty \quad \text{when} \quad \beta \to \pi, \ \alpha \to 0.$$

Toroidal coordinates introduced by equations analogous to (1), in which the radius $R_0$ of the disk is replaced by the radius $a$ of the drop, however, are used here. The solution of the problem formulated is obtained similarly to the solution of the preceding problem and has the form

$$\rho'' = \rho''_\infty - \sqrt{2(\operatorname{ch}\alpha + \cos\beta}\,\Delta\rho'' \int_0^\infty \frac{\operatorname{ch}[\tau(\pi-\beta)]}{\operatorname{ch}^2(\pi\tau)}\, p_{-1/2+i\tau}(\operatorname{ch}\alpha)\, d\tau.$$

The total flux of vapor to the drop is

$$G_1 = 2\pi D\int_0^\infty \left(R\frac{\partial\rho}{\partial\beta}\right)_{\beta=0} d\alpha = 2Da\Delta\rho''. \tag{11}$$

The growth rate of such a drop is

$$\frac{da}{dt} = \frac{2D\Delta\rho''\sin^3\beta_0}{\pi a(1-\cos\beta_0)^2(2+\cos\beta_0)\rho'}.$$

Thus, we obtain an equation which is simultaneously an upper estimate for the variation in the dimensions of a drop of condensate sufficiently distant from the boundary of the disk for all possible modes of dropwise condensation which proceed without the effective influence of free convection:

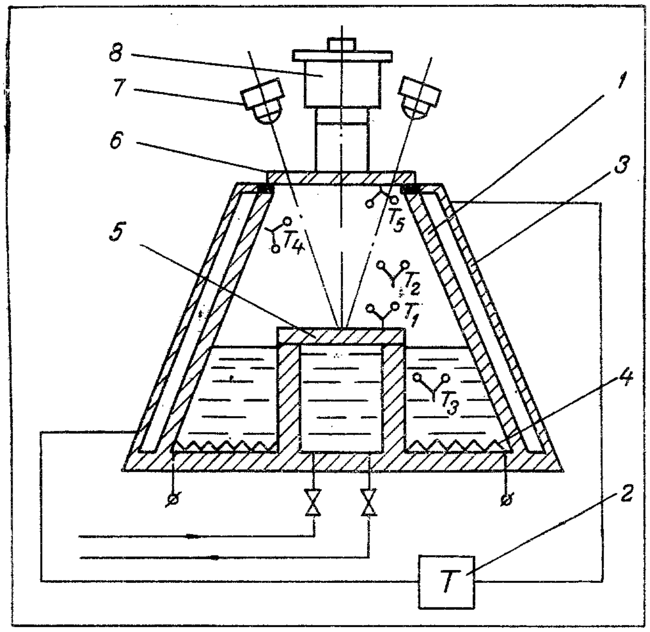

Fig. 3. Schematic diagram of experimental apparatus.

$$\Delta a^2 = \frac{4D\Delta\rho'' \sin^3\beta_0}{\pi\rho' (1 - \cos\beta_0)^2 (2 + \cos\beta_0)} \Delta t.$$

In all of the above we have neglected the thermal resistance of the drop, assuming that the role of this resistance in the process under consideration is negligibly small compared with the role of the diffusion resistance of the vapor–gas medium. In order to justify this assumption let us use an obvious upper estimate for the temperature differential $\Delta T$ in a drop of condensate. It is obvious that the heat flux corresponding to the vapor flux $G_1$ to the drop is

$$Q = rG_1 > \pi a^2 \lambda \Delta T / h, \quad (12)$$

where $h = a \tan(\beta_0/2)$ is the height of the spherical segment corresponding to the drop (Fig. 1b). We will compare this temperature differential $\Delta T$ with the value $\Delta\rho''$ by introducing the dimensionless parameter $\varepsilon = \Delta T \rho''_T / \Delta\rho''$, where $\rho''_T = \partial\rho''/\partial T$ ($[\rho''(T)]$ is the density of the saturated vapor at the temperature T in accordance with the Clausius–Clapeyron law). For the comparison we take the case of the minimum diffusion resistance corresponding to the kinetics of the growth of a drop sufficiently distant from the boundary of the disk, where $G_1$ is estimated from Eq. (11). In this case we obtain the following estimate from (12):

$$\varepsilon < \frac{2Dr\rho''_T \,\mathrm{tg}\,\beta_0/2}{\pi\lambda}.$$

But $Dr\rho''_T/\lambda \ll 1$ and, consequently, $\varepsilon \ll 1$, at least if the boundary angle $\beta_0$ is not too close to the value $\pi$.

To test the validity of the equations obtained and the assumptions made in the process, experimental studies were conducted on the determination of the growth rate of drops of condensate on a disk placed horizontally. A schematic diagram of the apparatus on which the tests were conducted is presented in Fig. 3.

The apparatus consists of a thermally regulated housing 1 in which the assigned constant temperature of the medium of humid air ($T = T_\infty$) and the vapor concentration corresponding to this temperature ($\rho'' = \rho''_\infty$) were maintained. The housing was thermally regulated by water from the thermostat 2 flowing in the loop formed by the housing and the outer casing 3.

The air was humidified by the evaporation of distilled water maintained in the constant-temperature housing at the temperature $T = T_\infty$. The heat needed to evaporate the water was supplied with the electric heater 4.

The condensation of vapor from the humid air took place on a copper disk 5 cooled by water circulating in a cavity under the disk. Oleic acid, which was spread in a thin layer on the surface of the copper disk, was used as a water repellant.

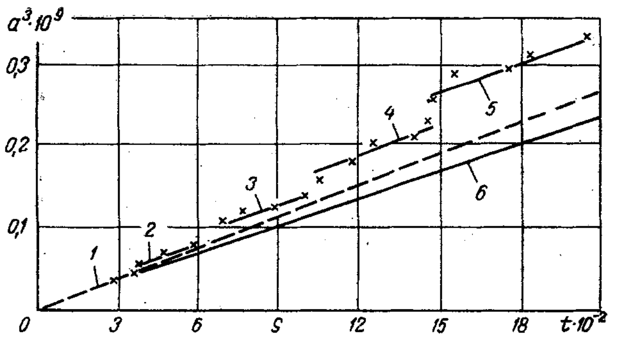


Fig. 4. Growth rate of drops of condensate when $\Delta\rho'' = 0.023$ kg/m$^3$ and $R/R_0 = 0.666$; $a$, m; t, sec.

A viewing window of optical glass 6 was provided on top of the constant-temperature housing for visual observation and motion-picture photography of the process of formation and growth of drops of condensate. A UIM-21 instrumental microscope 8 through which the visual observations and motion-picture photography of the condensation process were conducted was mounted at a fixed distance from the viewing window. The photography was conducted in reflected light. Two motion-picture projector lamps 7 placed at a fixed angle to the cooled surface were used for illumination.

The temperatures for the humid air ($T_2$), the cooled surface ($T_1$), the evaporating liquid ($T_3$), the walls of the thermally regulated housing ($T_4$), and the optical glass ($T_5$) were measured with thermocouples and controlled during the experiments.

The microscopic observations and motion-picture photography showed that in the initial stage of the condensation process the number of liquid drops on the disk is very large, and therefore their intensive merging with one another takes place owing to capillary forces. The zone of the phase transition moves away from the surface of the disk in proportion to the increase in the sizes of the liquid drops, and therefore the principal share of the vapor condenses on the large drops. Directly at the surface of the disk the concentration of vapor in the medium is small or insufficient to overcome the energy barrier connected with the formation of the surface of phase separation. Because of this the formation of new nuclei of the liquid phase around the large drops does not occur as intensively as in the initial stage of the condensation process. In addition, the smaller drops which form are actively "attracted" by the large drops and merge with them. One can explain the mechanism of such "attraction" on the basis of the dependence of the surface tension coefficient $\sigma$ on the temperature. For almost all liquids $\partial\sigma/\partial T < 0$. Disturbances in the temperature field at the condensation surface are connected, on the one hand, with the uneven supply of vapor to this surface depending on its geometry and the thermodynamic environment near the disk and, on the other hand, with the local properties of the diffusion kinetics produced by the presence on the disk of relatively large and almost stationary drops. Each of these drops suppresses the supply of vapor to a certain neighborhood of the disk adjacent to it, thereby creating around itself a zone of reduced temperature into which the fine droplets stream, drawn by thermocapillary forces produced at the surface of gas – liquid phase transition in the presence on it of a temperature gradient. However, the "attraction" of the small drops to the larger ones takes place over certain time intervals during which one is able to trace the variation in the sizes of the most characteristic individual drops with the help of high-speed motion-picture.

The sizes of the drops of condensate were measured directly on the screen during viewing of the most characteristic motion-picture frames through a motion-picture projector. For this a coordinate grid was applied to the screen, also making it possible to observe a certain group of drops over a long period of time. The experimental data obtained on the growth rate of individual drops with different initial sizes when $\Delta\rho'' = 0.023$ kg/m$^3$ and $R/R_0 = 0.666$ are presented in Fig. 4.

As seen from the figure, the curves obtained for the growth rate of drops with different initial sizes have the same angle of inclination to the abscissa, i.e., the growth rate of the drops independent of their initial sizes, proves to be the same. By shifting the straight line segments 1-5, which approximate the experimental data, in the direction of the time axis one can also obtain the variation in the size of a drop when it does not merge with neighboring drops during the entire period of filming (Fig. 4, curve 6). The growth rate of a separate drop calculated from Eq. (10) is shown on the same figure with a dashed line. As seen from Fig. 4, the experimental data obtained are in rather good agreement with the calculated values.

The number of drops of condensate $N = N(R)$ located on a surface element of the disk was counted successively on individual frames of the film. As these measurements showed, the number of drops of condensate on a surface element of the disk remains almost constant during a time $\Delta t$ and depends only on the value $R/R_0$. The value of the contact wetting angle $\beta_0$ of the surface of the disk was determined by the graphic method. For this the dimensions of drop profiles were measured with a microscope. Then the drop profiles were drawn at an increased scale and the wetting angle $\beta_0$ between the tangent to the drop profile at the point of its contact with the solid surface, and the base of the drop was determined.

For surfaces of a copper disk treated with oleic acid the contact wetting angle was in the range of $\beta_0 = 30\text{-}35°$.

The variation in the growth rate of drops as a function of $R/R_0$ was also determined in the studies conducted. For this the filming of the process of drop growth was conducted at two values of $R/R_0$ (0.32 and 0.66). As the data obtained showed, Eq. (9) accounts for the variation in the growth rate of the drops with $R/R_0$ sufficiently correctly. In Fig. 2 the experimental points correspond to the growth rates of drops of condensate on a disk at the values $R/R_0 = 0.32$ and 0.66. Curves 1-3 correspond to the drop growth rates calculated from Eq. (9).

## NOTATION

$\rho''$, $\rho''_W$, $\rho''_\infty$ are the density of vapor, density of saturated vapor at temperature of cooled disk, and density of saturated vapor at temperature of medium surrounding disk ($kg/m^3$);

z, R are the coordinates.

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